Prof. Dale van Harlingen, UIUC, Physics 498 Superconducting Quantum Devices

Lecture 3: Models and theories of superconductivity

Start discussion of how to understand the superconducting state

Present four phenomenological approaches:

1. Thermodynamics picture --- motivated by early experiments

THEORIES OF SUPERCONDUCTIVITY

Phenomenological

Thermodynamics
Gorter-Casimir (two fluid)
London
Pippard
Ginzburg-Landau
Gorkov - Eliashberg (TDGL = Time-Dependent G-L)

Gorkov

Microscopic

BCS
+ Anderson
+ Bogoliubov
+ Midgal

Unconventional SC models

Which is right?

Which are useful?

macroscopic problems (E&M)
spatial variations (intermediate/mixed state)
current applied
macroscopic quantum phenomena

1^{st} principle calculations $\rightarrow Tc$
Thermodynamic details
Excitations --- tunneling
transport
electrodynamics

For superconductor device physics and quantum information applications? need some of both

Phenomena --- what were experiments saying?

(1) SC was new (and unusual) – perfect conductivity *and* perfect diamagnetism
(2) Sharp and reversible onset with temperature and magnetic field and current

$\Rightarrow$ New phase of matter with a phase transition

THERMODYNAMICS

Compare "energy" of N and S states $\Longrightarrow$ which thermodynamic energy is appropriate?

$\mathcal{U}$ = internal energy — OK for isolated systems

$F = \mathcal{U} - TS$ = Helmholtz free energy — OK for $\vec{B}$ = constant

Must account for energy from field source since $\vec{B}$ changes at transition (Meissner effect)

★ $G = \mathcal{U} - TS - PV - \vec{H} \cdot \vec{M}$ = Gibbs free energy

Compare N and S for same applied field and require that $G_N = G_S$ at the phase transition (i.e. at T_c or H_c)

THERMODYNAMICS

$$d\mathcal{U} = dQ - W$$

↑ internal energy ↑ heat input ↑ work done BY system

1[ST] Law of Thermodynamics

$$dQ = TdS$$

2[nd] Law of Thermodynamics (quasistatic) S= entropy

$$W = PdV - HdM$$

H = applied magnetic field

$$dU = TdS - PdV + H\,dM$$

$$G = \mathcal{U} - TS + PV - H\,M$$

$$dG = \boxed{d\mathcal{U}} - \boxed{TdS} - SdT + \boxed{PdV} + VdP - \boxed{H\,dM} - MdH$$

$$dG = -SdT + VdP - MdH$$

$$\boxed{dG = -MdH}$$ constant T, P

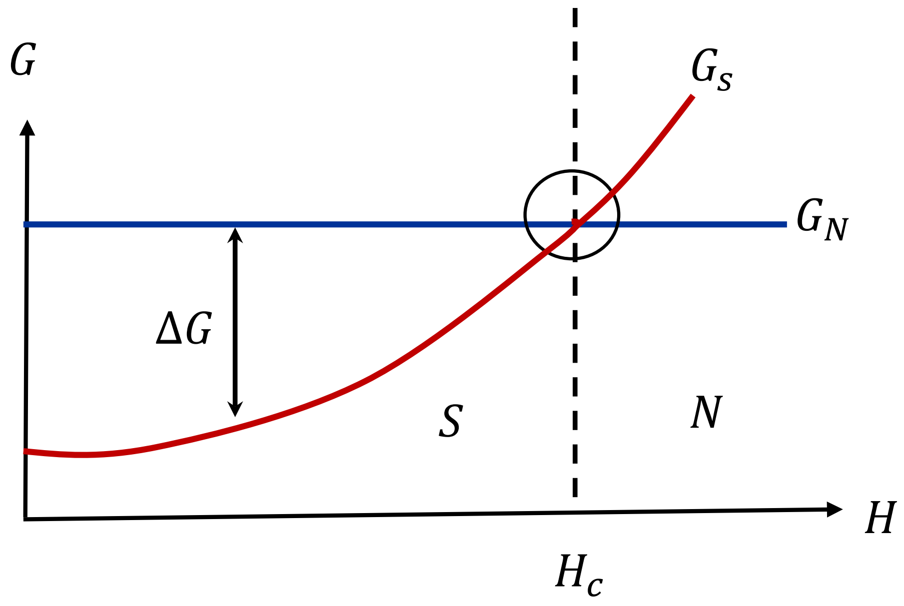
What do we know?

(1) Normal state: $G_N(H) = G_N(0)$ independent of magnetic field for a non-magnetic material

(2) Phase transition: $G_N(H_c) = G_S(H_c)$

(3) Superconducting state: $G_S(H) - G_S(0) = -\int_0^H M\,dH = -\frac{1}{4\pi}\int_0^H H\,dH = \frac{1}{8\pi}H^2$

$dG = -MdH$ $\qquad$ $M = -\frac{H}{4\pi}$ *perfect diamagnetism*



$$\Delta G(H) = G_N(H) - G_s(H)$$
$$= [G_n(0) - G_s(0)] - \frac{1}{8\pi}H^2$$
$$= \frac{1}{8\pi}(H_c^2 - H^2)$$

Depends on T

★ "Condensation energy" (stabilization energy) $\Delta G(0) = \frac{1}{8\pi}{H_c}^2$ $= \frac{1}{2}\mu_o {H_c}^2$ in MKS units

SC is a lower energy phase that is weakened by applying magnetic field --- costs energy to expel the field

Entropy

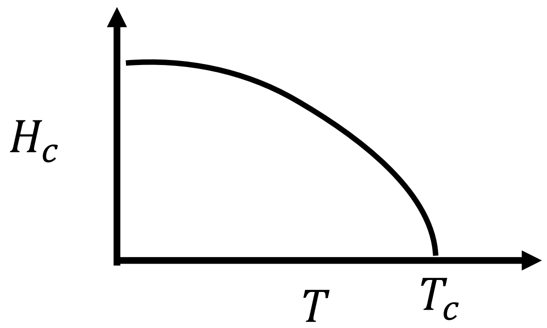
$$S = -\frac{\partial G}{\partial T}$$ for constant p, H

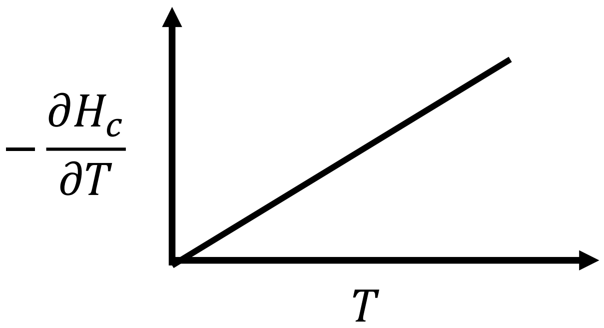
$$\Delta S = S_N - S_S = -\frac{\partial \Delta G}{\partial T} = -\frac{H_c}{4\pi}\left(\frac{\partial H_c}{\partial T}\right)$$ Independent of field

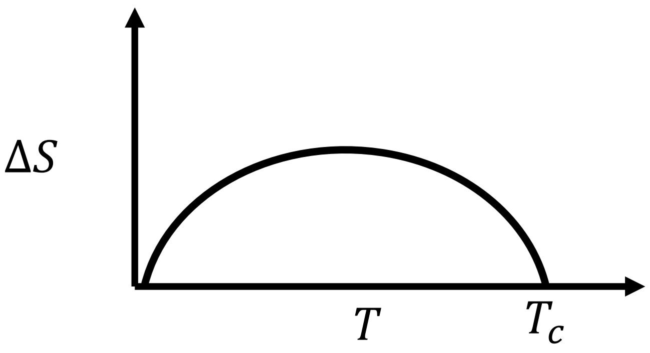
$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$-\frac{\partial H_c}{\partial T}(T) = \frac{2H_c(0)}{T_c}\left(\frac{T}{T_c}\right)$$

$$\Delta S = \frac{2H_c^2(0)}{T_c}\left(\frac{T}{T_c}\right)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

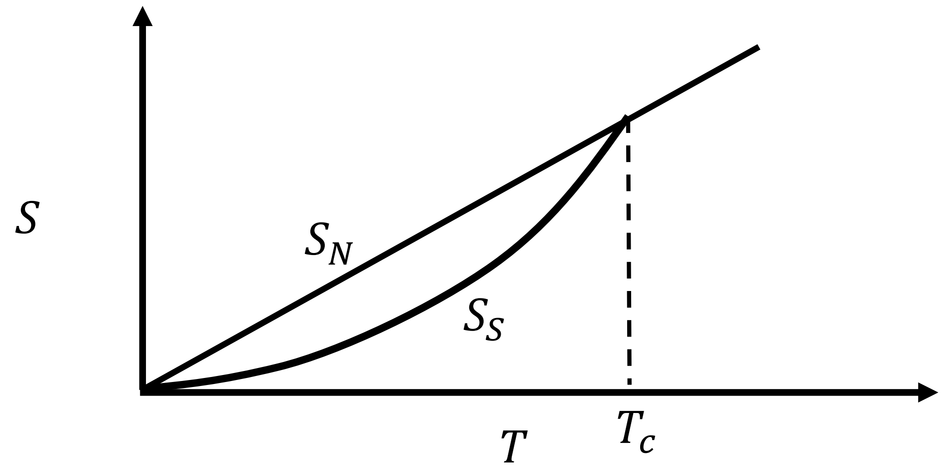






Normal state: $S_N(T) = \gamma T$ $\gamma = \frac{2}{3}\pi^2 N(0) k_B^2$ (Sommerfeld constant)

Superconducting state: $S_S(T) = S_S(T) - \Delta S$



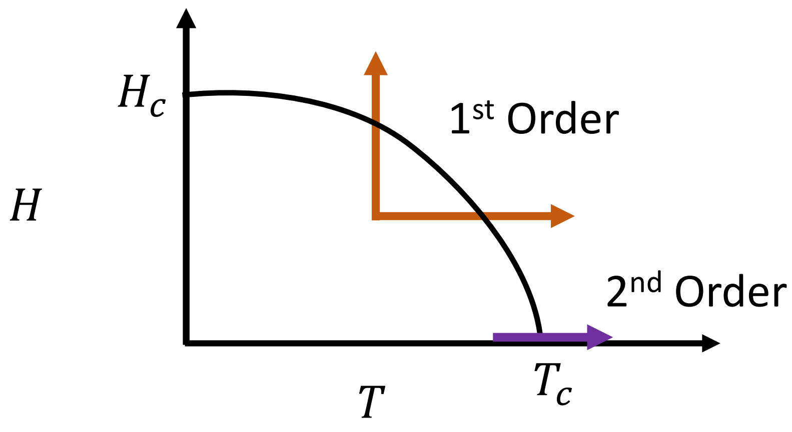
Comments: As $T \to 0$, $S \to 0$ (for both states) as required by the 3rd law $\Rightarrow$ $\frac{\partial H_c}{\partial T} = 0$

$S_S < S_N$ $\Rightarrow$ SC is an ordered state (BCS)

Slope $\frac{\partial S}{\partial T}$ discontinuous at $T = Tc$

Latent heat (at transition)

$$L = T\Delta S = -\frac{T H_c}{4\pi}\left(\frac{\partial H_c}{\partial T}\right)$$



- In zero magnetic field, no latent heat because the transition occurs at $T = T_c$, where $H_c = 0$
- In finite magnetic field for $T<T_c(H)$, transition by an applied field or an increase in T gives a finite latent heat

1st order in finite field (1st derivative of G is discontinuous)

2nd order phase transition in zero field (1st derivative of G is continuous)

Specific Heat

$$C = T\frac{\partial S}{\partial T}$$

$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

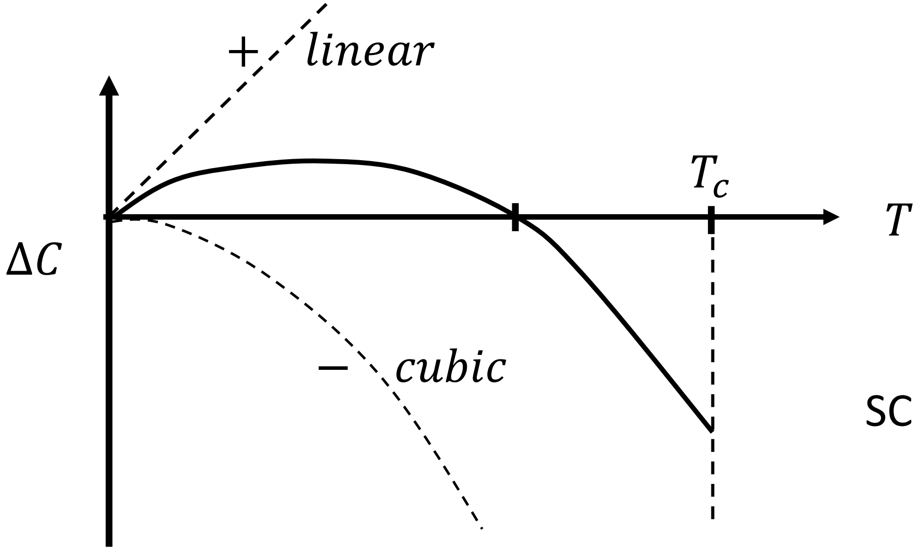
$$\Delta C = C_N - C_S = T\frac{\partial}{\partial T}\Delta S = -\frac{T}{4\pi}\frac{\partial}{\partial T}\left[H_c\left(\frac{\partial^2 H_c}{\partial T^2}\right)\right] = -\frac{1}{2\pi}\frac{H_c^2(0)}{T_c^2}T\left[2\left(\frac{T}{T_c}\right)^2 - 1\right]$$

- cubic +linear

$$\Delta C = 0 \quad \text{at} \quad T = 0$$

$$\Delta C = 0 \quad \text{at} \quad \mathrm{T} = \frac{1}{\sqrt{2}}T_c \approx 0.7T_c$$

$$\Delta C = -\frac{T_c}{4\pi}\left(\frac{\partial^2 H_c}{\partial T^2}\right)_{T_c} = -\frac{H_c^2(0)}{T_c} \quad \text{at} \quad T{=}T_c$$

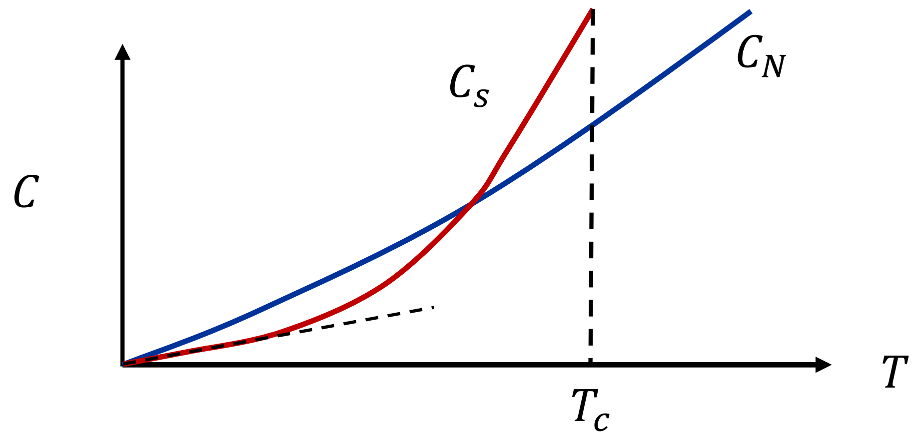


SC specific heat is higher

Normal state: $C_N(T) = aT + bT^3$

electrons phonons

Superconducting state: $C_S(T) = C_N(T) - \Delta C$



Thermodynamic predictions:

Linear at low T

Cubic at higher T

Jump at $T_c = \frac{1}{2\pi} \frac{H_c^2(o)}{T_c}$

Preview of BCS:

Exponential at low T $\Rightarrow$ energy gap

Jump precisely determined by $BCS = 1.43\gamma T_c$